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ON NECESSARY SAMPLE LENGTH TO DETERMINE POWER SPECTRUM.(U)
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(6) ON NECESSARY SAMPLE LENGTH TO DETERMINE POWER SPECTRUM.
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On Necessary Sample Length to Determine Power Spectrum....Reid D. Carlson

There is a characteristic property of an arbitrary signal that is useful to the design engineer. It is known as the power spectrum. Knowledge, for example, of the power spectrum of different types of noise would give an engineer necessary information to assist him in filtering it out of the response of his equipment.

However, given an infinite record of noise, it is necessary that the computer of the power spectrum know how much of the record he needs to analyze in order to give an accurate specification of the power spectrum.

It is the purpose of this memo to define a useful rule of thumb to prevent excessive labor on the part of the computer.

Let us take a finite section of our record so that it can be represented by the function $y(t)$, which is zero outside a time interval T and is identical to the noise record inside that interval.

Then $y(t)$ can be represented as the Fourier integral

1: $$y(t) = \int_{-\infty}^{\infty} A(f)\, e^{i\omega t}\, df$$

(Where $\omega = 2\pi f$)

Now, from the Parseval theorem, we have

2: $$\int_{-\infty}^{+\infty} y^2(t)\, dx = \int_{-\frac{T}{2}}^{\frac{T}{2}} y^2(t)\, dx = \int_{-\infty}^{+\infty} A^*(f)\, A(f)\, df$$

But,

3: $$A^*(f) = A(-f)\#$$

So,

4: $$A^*(f)\, A(f) = A(-f)\, A(f)$$

which is an even function of

#See Appendix

Thus,

5: $$\int_{-\frac{T}{2}}^{\frac{T}{2}} y^2(t)\,dt = 2\int_0^\infty |A(f)|^2\,df$$

Let us now consider the limit

6: $$\lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} y^2(t)\,dt = \lim_{T\to\infty}\int_0^\infty \frac{2\,|A(f)|^2\,df}{T}$$

The term on the left is the average power. The integrand of the integral on the right we shall call the power spectrum. Thus, if

7: $$\frac{2\,|A(f)|^2}{T} = G(f, T)$$

then we assume that $\lim_{T\to\infty} G(f,T)$ exists and call it the power spectrum.

Our problem is to determine a time T such that $G(f,T)$ ≈ power spectrum within our limits of error.

Now suppose we take larger and larger sections of our record (i.e., greater and greater T's) and we compute the average power for each. If we discover that the numbers associated with a given T no longer change appreciably by increasing T, then we can feel fairly confident that $G(f,T)$ is becoming constant with T (or we have approximated our power spectrum) because of the equality expressed by 6 above.

Thus, the following suggestion is made for the computation of power spectrum. Computations of the average power are made for increasing T until stability is reached, at which time the function $G(f,T)$ is computed and assumed to be an accurate representation of the power spectrum.

Appendix:

1: $$A(f) = \int_{-\infty}^{+\infty} y(t)\, e^{-i\omega t}\, dt$$

(where $\omega = 2\pi f$)

2: $$A^{*}(f) = \int_{-\infty}^{+\infty} y(t)\, e^{+i\omega t}\, dt$$

3: $$A(-f) = \int_{-\infty}^{+\infty} y(t)\, e^{+i\omega t}\, dt$$

So,

4: $$A^{*}(f) = A(-f)$$